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## Human Capital Investment And The Locally Rational Child

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**HUMAN CAPITAL INVESTMENT AND  
THE LOCALLY RATIONAL CHILD**

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## **HUMAN CAPITAL INVESTMENT AND THE LOCALLY RATIONAL CHILD\***

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### **ABSTRACT**

This study analyzes an overlapping generations economy with multiple family dynasties in which the ability levels of children are random and unobservable. Each parent allocates his income between consumption and investment in the education of his child. Each child, in turn, decides how much effort to exert in school on the basis of his perceived marginal returns to schooling, a perception shaped in part by the child's perception of the marginal returns to schooling attained by his parent. The income tax policies available to government range from libertarian (no redistribution) to perfectly egalitarian (equalization of after-tax family incomes in each period), where all income transfers are financed by current tax receipts. It is shown that policies resulting in small income transfers from richer to poorer families can increase social welfare by enabling poorer families to invest more optimally in their children, and by reducing uncertainty about future income. However, the taxes which fund the transfers also lower the expected marginal returns to schooling perceived by each child. As the tax policy becomes more egalitarian, the disincentive effects on schooling effort soon become dominant. In the limit, the perfectly egalitarian tax policy results in per capita income and social welfare levels that are well below those achieved under the libertarian tax policy. These adverse effects become more pronounced as the number of dynasties increases.

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## I. Introduction

The relationship between human capital and lifetime earnings is well-established in the economics literature. Particularly well-established is the positive link between increased years of education and increased real earnings. This link is apparent across all ethnic and racial groupings in the United States and, indeed, seems similarly well-established in other developed and lesser-developed countries [Schultz (1988, pp. 616-617)].

Unfortunately, absent public intervention, optimal investment in human capital is unlikely to occur. Applying the usual neoclassical principle, the Pareto optimal investment rule would be for each child to be educated up to the point where the present value of the investment costs are equal to the expected present value of future returns [Becker (1975), Rosen (1977)]. But investors in human capital are typically not able to use anticipated future earnings as collateral due to restrictions on indentured servitude and slavery.

Because of this capital market imperfection, investments in a child's human capital will generally be tied to the well-being of his parents rather than to his own innate skills and earnings capacity. If the costs of human capital investment are substantial, liquidity constraints on the poor will cause an underinvestment (from society's perspective) in the education of poor children. In addition, the suboptimal stock of human capital embodied in poor children will inflate the returns to human capital for wealthy children. As a consequence, there will be an overinvestment (from society's perspective) in the education of wealthy children.

This issue is discussed by Loury (1981) in his classic article on intergenerational transfers of wealth. Loury sets out a two-period lived overlapping generations model in which the ability levels of children are randomly distributed and unobservable. Each parent, concerned for the utility attained by his child, allocates his income between his own consumption and his investment in the human capital of his child. The public provision of education through purely redistributive nondistortionary tax policies is shown to improve social welfare

by removing the linkage between family income and educational investment. Subsequent researchers have generalized Loury's model in a number of directions. For example, Durlauf (1991) assumes that families endogenously organize themselves into neighborhoods, and that the amount of human capital that parents invest in their children is primarily determined by neighborhood characteristics.

An important issue remains to be addressed, however. Is the distortion in the educational investment in poorer children due solely to the absence of a human capital loan market, and hence to the unequal access of children to educational opportunity? Or can the distortion also be attributed in part to incentive problems that cause the children, themselves, to make inefficient use of the educational opportunities that they receive?

A commonly maintained assumption in traditional models of intergenerational transfers between parents and children is that the children are passive recipients of human capital investment.<sup>1</sup> Variations in human capital and educational attainments across children are attributed to differences in parental income and tastes and in children's endowments of ability or luck rather than to any choices made by the children themselves. In fact, however, children have considerable latitude in the amount of time and effort they devote to schooling. Children can select how many years to invest in schooling beyond their sixteenth year. Before that, children can vary the effort they expend on learning by truancy, shirking in school, homework, or extracurricular activities. Parents have only a partial influence on these choices.

A key factor influencing the amount of time and effort a child devotes to his schooling is the child's perceived returns to education [Becker (1975)]. Nondistortionary redistributive tax policies of the type analyzed by Loury (1981, p. 866) are unlikely to be feasible politically or administratively, as he himself points out. Rather, transfers to the poor are typically

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<sup>1</sup>Many recent endogenous growth studies [e.g., Lucas (1988), Tamura (1991), and Glomm and Ravikumar (1992)] assume that agents with direct control over physical resources invest in their own human capital in order to maximize their lifetime utility, with or without the additional support of a bequest. Here we retain the traditional assumption that children do not have any direct control over the physical resources devoted to their education.

funded by income taxes, and such taxes can distort individual choices. In particular, income taxes designed to equalize educational opportunity can cloud the perceptions of children regarding the returns to education per se, leading them to make inefficient use of educational opportunities.

This paper extends Loury (1981) in two principal ways. First, the child's decision regarding the effort he devotes to his schooling is explicitly incorporated into Loury's human capital investment problem. The child's effort in school is assumed to depend upon the child's perception of his marginal returns to schooling, which in turn is shaped by his perception of the marginal returns to schooling attained by his parent. Thus, "neighborhood effects" in the form of family influences have a direct impact upon the child's behavior.

Second, the treatment of government tax policy within our model has several novel features. Only distortionary income tax policies are available to government in its attempt to redistribute income from richer to poorer families. Also, the government is forced to satisfy a hard budget constraint in every period, in the sense that all distributed subsidies must be financed by current tax receipts. In contrast, Loury (1981, p. 853) only requires that government satisfy an *expected* budget constraint in making its period 1 plans for current and future periods. Finally, the tax policy in our model is characterized by a single parameter. As this parameter is increased from zero to an upper bound, the tax policy progresses from a purely libertarian policy with no redistribution of income to a perfectly egalitarian policy under which family incomes are equalized in each period. This permits us to analyze and compare welfare and income outcomes across a broad range of tax policies.

Our model demonstrates that simply equalizing educational opportunity, say through income transfers to poor families, will not necessarily result in Pareto optimal human capital investments. The crucial observation is that the government's tax policy can lower a child's perception of his marginal returns to schooling at the same time that it equalizes parental income transfers to children. We find, in particular, that a small degree of income

subsidization for low income families does increase human capital production, raising both social welfare and GNP. Yet social welfare and GNP both decline precipitously as government imposes progressively more egalitarian tax policies. Moreover, increasing the number of families in existence at each point in time exacerbates these disincentive effects.

The basic human capital investment model is set out in section II. A central way in which this model differs from Loury (1981) is in its inclusion of a behavioral relation describing the schooling effort of the child. In section III it is shown that this behavioral relation can be motivated using a model of human capital investment outlined in Becker (1975; pp. 94-144). In sections IV and V the dynamic properties of the basic human capital investment model are investigated for particular functional specifications chosen to match as closely as possible the specifications of Loury (1981) for comparison purposes. The results of extensive simulation experiments are reported in section VI. The final section VII gives concluding comments.

## **II. The Basic Economy**

This section sets out a model, referred to as the "Basic Economy," which generalizes the model used by Loury (1981) to investigate the relation between human capital investment, income distribution, and government tax and transfer policies. Loury assumes that the subsequent earnings of a child are determined by two factors: innate ability; and parental investment in the child's education. In the Basic Economy it is assumed that the child's subsequent earnings are also determined in part by the intensity of effort which the child devotes to his schooling.

Specifically, the Basic Economy is an overlapping generations (OG) economy which begins in period 1 and extends into the infinite future. The economy consists of a consumer sector and a government policymaker with tax and transfer powers. The rate of population growth is constant and equal to zero. Each agent lives for just two periods, a first period ("childhood") and a second period ("parenthood"). One child is born to each agent at the

beginning of his second period of life. The economy has only one consumable resource,  $Q$ , assumed to be completely perishable and divisible. Adopting the standard convention that goods are distinguished by date of availability, the amount of  $Q$  available during period  $t$  represents “good  $t$ .” The economy thus has an infinite number of goods.

In the initial period 1, the population consists of  $N$  parents and  $N$  children, divided into  $N$  parent-child pairings. The  $N$  pairings constitute  $N$  distinct “family dynasties,”  $i = 1, \dots, N$ . Each child born in a subsequent period  $t$  is then assigned to one of the  $N$  family dynasties on the basis of the dynasty belonged to by his parent.

Without loss of generality, attention will be focused in this section on any one family dynasty, say dynasty  $i$ . Dynasty  $i$  in the initial period 1 consists of one parent,  $P_{i1}$ , together with his child  $C_{i1}$ . The parent  $P_{i1}$  is assumed to have an exogenously given amount of pre-tax earnings  $y_{i1}$  measured in period 1 good, and is also characterized by an exogenously given intensity of effort level  $n_{i0}$  representing the effort which he put into his schooling as a child. In each subsequent period  $t \geq 2$ , dynasty  $i$  then consists of one parent  $P_{it}$ , born in period  $t - 1$ , together with the child  $C_{it}$  of  $P_{it}$  born at the beginning of period  $t$ .

In each period  $t \geq 1$ , the child  $C_{it}$  is endowed with an unobservable random skill level  $\alpha_{it}$  for producing  $Q$ , where  $\alpha_{it} \in [0, 1]$ . In addition, his parent  $P_{it}$  might choose to invest a certain amount of resources  $e_{it}$  in his education, where  $e_{it}$  is measured in period  $t$  good. Finally, the child himself must decide how much intensity of effort  $n_{it}$  he wishes to devote in school to his education. The level of pre-tax earnings  $y_{i,t+1}$  achieved by the child  $C_{it}$  when he reaches adulthood in the subsequent period  $t + 1$  is assumed to depend positively on all three factors. Formally, then the child’s income (or human capital production) function takes the form

$$y_{i,t+1} = h(\alpha_{it}, e_{it}, n_{it}), \quad h_1 > 0, \quad h_2 > 0, \quad h_3 > 0, \quad (1)$$

where  $y_{i,t+1}$  is measured in good  $t + 1$ .

Suppose the government policymaker at the beginning of period 1 selects a tax policy



$\tau$  for determining the taxes and subsidies to be imposed on the earnings of each parent in each subsequent period. Let  $\tau_{it}$  denote the tax or subsidy level imposed on the dynasty  $i$  parent  $P_{it}$  in period  $t$  under tax policy  $\tau$ , where  $\tau_{it} < 1$  denotes a tax and  $\tau_{it} > 1$  denotes a subsidy (or "negative tax"). The after-tax earnings of  $P_{it}$  then take the form  $\tau_{it}y_{it}$ . It will be supposed that  $\tau_{it}$  is determined as some function  $G(\cdot)$  of the pre-tax earned incomes  $y_{jt}$ ,  $j = 1, \dots, N$ , for the  $N$  dynasties in period  $t$ , the tax policy  $\tau$ , and the total number of dynasties  $N$ . That is,

$$\tau_{it} = G_i(y_{1t}, \dots, y_{Nt}, \tau, N) . \quad (2)$$

The child  $C_{it}$  must decide on the intensity of effort  $n_{it}$  he devotes to his schooling. In accordance with numerous empirical studies, it will be assumed that  $C_{it}$  is positively influenced in this decision by two factors: (a) the amount of resources  $e_{it}$  which his parent invests in his education; and (b) his expected rate of return to schooling effort, denoted by  $r_{it}^e$ , where

$$r_{it}^e = (\partial \tau_{i,t+1} y_{i,t+1} / \partial n_{it})^e . \quad (3)$$

Formally, the intensity of effort  $n_{it}$  takes the functional form<sup>2</sup>

$$n_{it} = n(e_{it}, r_{it}^e) , \quad n_1 > 0, \quad n_2 > 0 . \quad (4)$$

It follows from (4) that a parent can influence the education of his child in two different ways. First, a parent can directly transfer resources to his child through the human capital investment level  $e_{it}$ . The investment level  $e_{it}$  reflects the extent of government taxes and transfers, since it is allocated out of disposable income. Second, as will be clarified below, the information the child obtains about his parent's income and effort levels can influence the child's expected rate of return to schooling effort.

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<sup>2</sup>In section III, we provide additional motivation for the functional specification (4) by deriving it from certain fundamental cost-benefit relations detailed in Becker (1975, pp. 94-144).

The parent  $P_{it}$  faces a more complicated decision problem in period  $t$  than his child. Specifically, he must decide how much of his disposable income to devote to his own consumption versus how much to devote to investment in the education of his child. The utility attained by  $P_{it}$  in period  $t$  is assumed to be a function  $U(c_{it}, V_{i,t+1})$  of his own period  $t$  consumption,  $c_{it}$ , and the utility  $V_{i,t+1}$  he anticipates for his child in period  $t + 1$ .

Let  $s_{it}$  denote the state vector describing the situation of the parent  $P_{it}$  at the beginning of period  $t$  in terms of his pre-tax earnings  $y_{it}$ , his past schooling effort  $n_{i,t-1}$ , and his income tax or subsidy level  $\tau_{it}$ , as well as the government's tax policy  $\tau$  and the total number of dynasties  $N$ . That is, define

$$s_{it} = (y_{it}, n_{i,t-1}, \tau_{it}, \tau, N) . \quad (5)$$

The state vector (5) represents the information which is potentially available to the child  $C_{it}$  in period  $t$ . Consequently, it seems reasonable to suppose that the child's expected rate of return to schooling effort, described in (3), is determined as some function

$$r_{it}^e = r^e(s_{it}) \quad (6)$$

of the state vector (5).

It follows from (6) that the child's expectations regarding returns to schooling are potentially affected by the tax policy  $\tau$ , as well as his family's particular tax or transfer level  $\tau_{it}$ . Thus, the tax system can influence the child's schooling effort—i.e., his human capital investment decision—by distorting the child's perceived returns to schooling. In contrast, in Loury (1981) the schooling effort of the child is effectively held constant, implying that the tax system cannot influence the human capital investment decision of the child. The importance of the potential influence of the tax system on human capital investment by children is illustrated for a special case of the Basic Economy in section IV, below.

Given (1), (4), and (6), the state vector  $s_{it}$  satisfies a recurrence relation of the form

$$s_{i,t+1} = ( h(\alpha_{it}, e_{it}, n(e_{it}, r^e(s_{it})), n(e_{it}, r^e(s_{it})), \tau_{i,t+1}, \tau, N) \quad (7)$$

$$\equiv F(\alpha_{it}, e_{it}, \tau_{i,t+1}, s_{it}) .$$

The budget set facing  $P_{it}$  in period  $t$  takes the form

$$B(s_{it}) = \{(c_{it}, e_{it}) \geq 0 \mid c_{it} + e_{it} = \tau_{it}y_{it}\} . \quad (8)$$

The “value function”  $V_t(s_{it})$  for the parent  $P_{it}$  is then defined to be the maximum expected utility attainable by  $P_{it}$  in period  $t$ , given the budget set  $B(s_{it})$ . By construction, this value function satisfies a dynamic programming recurrence relation of the form

$$V_t(s_{it}) = \max_{c_{it}, e_{it} \in B(s_{it})} E[U(c_{it}, V_{t+1}(s_{i,t+1})) \mid s_{it}] , \quad (9)$$

subject to  $s_{i,t+1}$  being given by relation (7). The  $s_{it}$ -conditional expectation in (9) is taken with respect to the ability level  $\alpha_{it}$  and the tax rate  $\tau_{i,t+1}$  appearing in (7). Note that this construction implies that the parent knows the income function  $h(\cdot)$  and the function  $n(\cdot)$  which determines the intensity of effort levels of children.

A central way in which our model differs from that of Loury (1981) is the introduction of the behavioral relation (4) describing the schooling effort of the child. It is therefore important to provide additional motivation for this feature of the model. In the next section it is shown that relation (4) is consistent with the view that the child makes rational use of limited information to decide on the allocation of his time during his schooling years. Thus, in contrast with other recent studies of human capital investment [e.g., Glomm and Ravikumar (1992)], the child solves a local optimization problem involving his current schooling experience rather than a global optimization problem involving the intertemporal allocation of physical resources and a concern for future generations.

### III. Factors Determining a Child's Optimal Schooling Effort

In his Woytinsky lecture, Becker (1975, pp. 94-144) outlines a model in which individual human capital investment and the return to this investment are simultaneously determined

by a downward sloping curve depicting the marginal return to education and an upward sloping curve depicting the marginal cost of financing education. Here we adapt Becker's model to the determination of how much effort a child will expend in school.

Let  $n_t$  represent the effort a child expends in school. Effort represents the proportion of time the child actively invests in learning, e.g., by paying attention in school, studying, reading, and so forth. While truancy laws may require that all children spend time in school, this time only produces human capital when the child is concentrating on learning. Using Leibowitz' (1974) terms, truancy laws have tended to equalize human capital investment at the extensive margin (years of school); but there is still a considerable range of choice regarding the intensity of investment as measured by class attendance, homework completion, and extra-curricular learning.

The marginal return to schooling effort of a child in period  $t$ , denoted by  $MR^t$ , is given by the derivative of the child's adult after-tax earnings  $z_{t+1} = \tau_{t+1}y_{t+1}$  in period  $t + 1$  with respect to the child's schooling effort  $n_t$  in period  $t$ . This marginal return is assumed to be a decreasing function of the the child's schooling effort  $n_t$ , an increasing function of parental investment  $e_t$ , and an increasing function of the child's natural ability  $\alpha_t$ ; i.e.,

$$MR^t \equiv \partial z_{t+1} / \partial n_t = MR(n_t, e_t, \alpha_t), \quad MR_n < 0, \quad MR_e > 0, \quad MR_\alpha > 0. \quad (10)$$

The marginal cost to schooling effort of a child in period  $t$ , denoted by  $MC^t$ , is assumed to be an increasing function of schooling effort  $n_t$  and a decreasing function of parental investment  $e_t$ ; i.e.,

$$MC^t = MC(n_t, e_t), \quad MC_n > 0, \quad MC_e < 0. \quad (11)$$

The negative relationship between the marginal return  $MR^t$  and the level of schooling effort  $n_t$  is easily justified. The production of human capital is presumably subject to diminishing returns to all inputs, including effort. Consequently, the amount of human capital produced by successive incremental increases in effort will tend to decline. This in turn

implies that the earnings generated from successive incremental increases in effort will tend to decline. As noted by Becker (1975, p. 98), diminishing returns to human capital investment are a necessary consequence of the fact that human capital must be embodied in the individual doing the investing.

The positive relationship between the marginal cost  $MC^t$  and the level of schooling effort  $n_t$  can be justified on the basis that the child obtains utility from leisure time. In this case, schooling effort has an hedonic cost associated with it. If utility from leisure is subject to diminishing marginal returns, then the marginal hedonic cost of schooling effort will rise with increased schooling effort (decreased leisure).

The assumption that marginal return  $MR^t$  and marginal cost  $MC^t$  increase and decrease, respectively, with an increase in the amount of parental investment  $e_t$  in the child's education can be justified on the basis that the child has more material resources to work with for any given level of ability and schooling effort. This both improves his productivity (increases his marginal return) and makes schooling easier (decreases his marginal cost). Finally, an increase in the child's natural ability  $\alpha_t$  is plausibly assumed to result in an increase in  $MR^t$  for any given level of parental investment and schooling effort, since  $\alpha_t$  is defined to be a measure of the child's innate productive capacity.

The child is assumed to choose his optimal effort level,  $n_t^*$ , by equating his marginal cost to his expected marginal return; i.e.,

$$MC^t = MC(n_t, e_t) = (MR^t)^e . \quad (12)$$

The optimal effort level will differ across children depending upon the level of parental investment  $e_t$  and the child's natural ability  $\alpha_t$ . As depicted in Figure 1, the marginal return curve shifts up and the marginal cost curve shifts down if parental investment increases from  $e_t$  to  $e'_t$ . As a result, the optimal effort level  $n_t^*$  increases with increases in  $e_t$ , although the marginal return to effort may either rise or fall.

—Figure 1 About Here—

Consequently, it follows from (12) that the child's optimal effort level  $n_t^*$  takes the form

$$n_t^* = n(e_t, (MR^t)^e), \quad (13)$$

where  $n(\cdot)$  is an increasing function of both parental investment  $e_t$  and expected marginal return to schooling effort  $(MR^t)^e$ . The correspondence with the assumed relation (4) for the child's schooling effort is immediate.

#### IV. An Illustrative Special Case of the Basic Economy

In this section, the dynamic properties of the Basic Economy are investigated for particular specifications for the income function, the intensity of effort function, the utility function, and the tax policy set out in general form in section II. These specifications are chosen to match as closely as possible the example studied by Loury (1981, pp. 855-857).

Consider, then, a special case of the Basic Economy in which the income function (1) for each dynasty  $i$  in each period  $t$  is given by

$$\begin{aligned} y_{i,t+1} &= h(\alpha_{it}, e_{it}, n_{it}) \\ &= \lambda \cdot (\alpha_{it})^m \cdot (e_{it})^u \cdot (n_{it})^v \end{aligned} \quad (14)$$

for arbitrary positive constants  $\lambda$ ,  $m$ ,  $u$ , and  $v$  with  $0 < (u + v) \leq 1$ . Suppose, also, that the intensity of effort function (4) for each dynasty  $i$  in each period  $t$  is given by

$$\begin{aligned} n_{it} &= n(e_{it}, r^e(s_{it})) \\ &= (e_{it})^a \cdot (r^e(s_{it}))^b \end{aligned} \quad (15)$$

for arbitrary positive constants  $a$  and  $b$  satisfying  $0 < (a + b) \leq 1$ .

The utility attained by each dynasty  $i$  parent  $P_{it}$  in each period  $t$  is assumed to be given by

$$U_{it} = (c_{it})^\gamma \cdot (V_{i,t+1})^{1-\gamma}, \quad (16)$$

where  $c_{it}$  denotes the period  $t$  consumption of  $P_{it}$ ,  $V_{i,t+1}$  denotes the utility which  $P_{it}$  anticipates for his child in period  $t + 1$ , and  $\gamma$  is an arbitrary constant assumed to satisfy  $0 < \gamma < 1$ .<sup>3</sup> Suppose  $(1 - \gamma)^k V_{i,t+k}$  remains bounded as  $k$  becomes arbitrarily large. If anticipations are correct, it then follows that

$$\begin{aligned}
 \log(U_{it}) &= \gamma \log(c_{it}) + (1 - \gamma) \log V_{i,t+1} \\
 &= \gamma \log(c_{it}) + (1 - \gamma)[\gamma \log(c_{i,t+1}) + (1 - \gamma)V_{i,t+2}] \\
 &\vdots \\
 &= \gamma \left[ \sum_{k=0}^{\infty} (1 - \gamma)^k \log(c_{i,t+k}) \right].
 \end{aligned} \tag{17}$$

Consequently, the ex post “true” utility of each dynasty  $i$  parent  $P_{it}$  in each period  $t$  is a function of the consumption levels of all current and future members of dynasty  $i$ .

The government must select a tax policy which is budgetarily feasible. Since the resource  $Q$  for the Basic Economy is nonstorable, budgetary feasibility in each period  $t$  requires that the sum of the after-tax incomes of the  $N$  dynasties should not exceed the sum of their before-tax incomes.<sup>4</sup> Letting  $\tau_{it}$  denote the tax or subsidy imposed on the dynasty  $i$  parent  $P_{it}$  in period  $t$ , the condition expressing this feasibility is

$$\sum_{i=1}^N \tau_{it} y_{it} \leq \sum_{i=1}^N y_{it}. \tag{18}$$

It will be supposed that the government selects a tax policy from among a family of feasible tax policies parameterized by  $\tau$ , where  $\tau$  lies in the interval  $[0, 1/N]$ . Specifically, given any  $\tau$  in  $[0, 1/N]$ , and given the pre-tax income levels  $\{y_{it} : i = 1, \dots, N\}$  of the  $N$  dynasties in period  $t$ , the tax or subsidy level  $\tau_{it}$  imposed on each parent  $P_{it}$  is determined

<sup>3</sup>If  $\gamma = 1$ , parents place no weight on their progeny, and hence do not invest in education. Given (14), this in turn dooms all future generations to have zero income.

<sup>4</sup>Loury (1981, Def. 3, p. 853) calls a tax policy “purely redistributive” if it satisfies such a government budget constraint *in expectation*. However, Loury does not require that government *actually* satisfy its budget constraint in each period  $t$ , as is imposed here.

by the relation

$$\tau_{it}y_{it} \equiv [1 - (N - 1)\tau] \cdot y_{it} + \tau \cdot \left( \sum_{j \neq i} y_{jt} \right). \quad (19)$$

First note that the tax policies  $\tau$  in  $[0, 1/N]$  all satisfy the budget feasibility condition (18) with exact equality, by construction. Moreover, the tax policies range from “libertarian” to “egalitarian” as  $\tau$  ranges from 0 to  $1/N$ . Given the tax policy  $\tau = 0$ , the tax rates  $\tau_{it}$  are identically equal to 1, implying that no redistribution is undertaken. On the other hand, the tax policy  $\tau = 1/N$  is egalitarian in the sense that each of the  $N$  dynasties in each period  $t$  receives precisely  $1/N$  of the total income earnings of the  $N$  dynasties. Given any existing configuration of pre-tax incomes, the tax policy becomes progressively more egalitarian as  $\tau$  ranges from 0 to  $1/N$ , in the sense that the variance of after-tax incomes declines.

More can now be said about  $r_{it}^e$ , the marginal return to schooling effort anticipated by the dynasty  $i$  child  $C_{it}$  in period  $t$ . Recall from section II, relation (3), that  $r_{it}^e$  is assumed to be measured in terms of  $C_{it}$ 's after-tax income earnings for period  $t + 1$ . Let  $r_{it}$  denote the actual marginal return to schooling effort attained by  $C_{it}$  in the subsequent period  $t + 1$ . Using (14) and (19), one has

$$\begin{aligned} r_{it} &= \partial \tau_{i,t+1} y_{i,t+1} / \partial n_{it} \\ &= \partial [1 - (N - 1)\tau] y_{i,t+1} / \partial n_{it} \\ &= [1 - (N - 1)\tau] \partial y_{i,t+1} / \partial n_{it} \\ &= [1 - (N - 1)\tau] v \cdot [y_{i,t+1} / n_{it}]. \end{aligned} \quad (20)$$

Thus, the *marginal* return to schooling effort,  $r_{it}$ , is proportional to the *average* return to schooling effort,  $y_{i,t+1}/n_{it}$ . Note that  $[1 - (N - 1)\tau]$  is nonnegative for all  $\tau$  in the admissible range  $[0, 1/N]$ .

In section II,  $r_{it}^e$  is assumed to be a function  $r^e(s_{it})$  of the state vector  $s_{it}$  consisting of the information which is potentially available to the child  $C_{it}$  in period  $t$ . In keeping with this assumption, it will be supposed here that the child estimates his own average return to



schooling effort,  $y_{i,t+1}/n_{it}$ , by considering the average return to schooling effort attained by his parent,  $y_{it}/n_{i,t-1}$ . Thus,<sup>5</sup>

$$\begin{aligned} r_{it}^e &= r^e(s_{it}) \\ &\equiv [1 - (N - 1)\tau]v \cdot [y_{it}/n_{i,t-1}] . \end{aligned} \quad (21)$$

It follows from (21) that, for *each* child, the expected marginal return to schooling effort declines with increases in  $\tau$ , i.e., with increases in the degree of egalitarianism of the government's tax policy. As will be clarified in section VI, this negative effect can offset the positive effect of income transfers on human capital investment.

Finally, restrictions will be placed on the stochastic properties of the model. First, it will be supposed that the random ability levels of children across the dynasties  $i = 1, \dots, N$  are governed by stationary independent probability distributions ( $f_1(\cdot), \dots, f_N(\cdot)$ ) in each period  $t$ . Second, it will be supposed that each dynasty  $i$  parent believes he is unable, by his own actions, to affect the tax or subsidy level to be imposed on his child. More precisely, given any function  $J(\tau_{i,t+1})$  of  $\tau_{i,t+1}$ , it will be supposed that the  $s_{it}$ -conditioned expected value of  $J(\tau_{i,t+1})$  formed by the parent  $P_{it}$  takes the form

$$E[J(\tau_{i,t+1}) | s_{it}] = E[J(\tau_{i,t+1}) | \tau_{it}, \tau, N] . \quad (22)$$

Thus,  $P_{it}$  believes that the information in the state vector  $s_{it} = (y_{it}, n_{i,t-1}, \tau_{it}, \tau, N)$  regarding his own pre-tax earnings  $y_{it}$  and his own intensity of schooling effort  $n_{i,t-1}$  is not relevant for estimating the tax or subsidy level  $\tau_{i,t+1}$  to be imposed on his child. Essentially, this reduces to assuming that each parent views himself as being too small to affect government tax policy.

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<sup>5</sup>This expectational assumption is consistent with the views of Murray (1984), Wilson (1987), Case and Katz (1991), and Streufert (1991), among others, who argue that children can have clouded perceptions of the true returns to schooling because their primary source of information is their own immediate family and neighborhood. Nevertheless, as argued by Manski (1991), more empirical work is needed to learn how children actually do infer their returns to schooling.

## V. Dynamic Properties of the Illustrative Basic Economy

Given the functional forms specified in section IV, this section proves a number of propositions for the Basic Economy. It is demonstrated that for all tax policies, the parent will invest some positive amount in his child's human capital. This holds even under the perfectly egalitarian tax policy. However, in the long run, the expected income and schooling effort levels of each dynasty are strictly lower under the egalitarian than under the libertarian tax policy. Furthermore, the gap between the egalitarian and libertarian outcomes increases as the number of dynasties increases. The reason is that the expected long-run marginal return to schooling effort decreases unambiguously as the number of dynasties increases in the egalitarian economy, but is unaffected by the number of dynasties in the libertarian economy. Thus, the economy that attempts to redistribute income equally among all of its citizens is plagued by free-rider effects that drive expected income and schooling effort toward zero as the number of dynasties increases.

Formal statements of these and other propositions are presented below, and proofs are given in the Appendix. The first proposition provides a more concrete analytical representation for the recurrence relation satisfied by the value function  $V_t(s_{it})$ .

**PROPOSITION V.1.** *Suppose the value function for the child  $C_{it}$  upon reaching adulthood in period  $t + 1$  can be expressed in the form*

$$V_{t+1}(s_{i,t+1}) = (y_{i,t+1})^{\delta_{t+1}} \cdot (n_{it})^{-\theta_{t+1}} \cdot K(\beta_{t+1}, g_{i,t+1}) \quad (23)$$

*for some nonnegative coefficient vector  $\beta_{t+1} \equiv (\delta_{t+1}, \theta_{t+1})'$  and some function  $K(\beta_{t+1}, g_{i,t+1})$  of  $\beta_{t+1}$  and  $g_{i,t+1} \equiv (\tau_{i,t+1}, \tau, N)$ . Then the value function for the parent  $P_{it}$  of  $C_{it}$  in period  $t$  takes the form*

$$V_t(s_{it}) = (y_{it})^{\delta_t} \cdot (n_{i,t-1})^{-\theta_t} \cdot K(\beta_t, g_{it}) , \quad (24)$$

*where  $\beta_t \equiv (\delta_t, \theta_t)'$  is a nonnegative coefficient vector which satisfies a matrix recurrence*

relation of the form

$$\beta_t = \gamma\pi + (1 - \gamma)M\beta_{t+1} , \quad (25)$$

and  $K(\beta_t, g_{it})$  satisfies a recurrence relation of the form

$$K(\beta_t, g_{it}) = F(\beta_{t+1}, g_{it}) \cdot E[K(\beta_{t+1}, g_{i,t+1})^{1-\gamma} | g_{it}] . \quad (26)$$

As a corollary to Proposition V.1, it can be shown that the period  $t$  optimal consumption and investment levels for the dynasty  $i$  parent are proportional to his period  $t$  after-tax income.

**COROLLARY V.1:** *Under the assumptions of Proposition V.1, the parent  $P_{it}$ 's optimal choices for human capital investment  $e_{it}$  and consumption  $c_{it}$  take the form*

$$e_{it}^o = \tau_{it}y_{it} \cdot (1 - \gamma)w_{t+1}/[\gamma + (1 - \gamma)w_{t+1}] ; \quad (27)$$

$$c_{it}^o = \tau_{it}y_{it} \cdot \gamma/[\gamma + (1 - \gamma)w_{t+1}] , \quad (28)$$

where  $w_{t+1} \equiv (u + av)\delta_{t+1} - a\theta_{t+1}$ ,  $t \geq 1$ .

As indicated by Proposition V.1 and Corollary V.1, both the value function and the optimal consumption and investment decisions of the dynasty  $i$  parent have a nonstationary structure if the coefficient vector  $\beta_t = (\delta_t, \theta_t)$  that parameterizes the value function (24) changes over time. As established in the next Proposition V.2, however, the recurrence relation for  $\beta_t$  has a unique stationary solution.

**PROPOSITION V.2:** *The recurrence relation (25) has a unique stationary solution  $\bar{\beta} = (\bar{\delta}, \bar{\theta})' > 0$  given by*

$$\bar{\delta} = \gamma[1 + (1 - \gamma)b]\Delta ; \quad (29)$$

$$\bar{\theta} = \gamma[(1 - \gamma)bv]/\Delta , \quad (30)$$

where  $\Delta > 0$  denotes the determinant of the matrix  $[I - (1 - \gamma)M]$ .

For easier comparison with Loury (1981), who focuses exclusively on stationary solutions, it will henceforth be assumed that the coefficient vector  $\beta_t$  coincides with  $\bar{\beta}$  for all  $t \geq 1$ . Combining Corollary V.1 and Proposition V.2, it is then straightforward to show that each parent's optimal choices (27) and (28) for human capital investment and consumption remain positive as long as he earns a positive after-tax income  $\tau_{it}y_{it}$ .

The next two propositions characterize the long-run behavior of dynasty income and intensity-of-effort levels under libertarian and egalitarian tax policies. For ease of notation, let

$$x_{it} = (\log(y_{it}), -\log(n_{i,t-1}))' \quad (31)$$

denote the indicated logarithmic transformation of the period- $t$  income and intensity-of-effort levels  $(y_{it}, n_{i,t-1})'$  for dynasty  $i$ . Also, let  $E[\cdot | s_1]$  denote an expectation conditional on the period 1 social state vector,

$$s_1 \equiv ((y_{11}, n_{10}), \dots, (y_{N1}, n_{N0}), \tau, N) . \quad (32)$$

The expectation is assumed to be taken with respect to the joint distribution for ability levels across dynasties over time.

For the libertarian tax policy  $\tau = 0$ , it follows from (19) that the after-tax income  $\tau_{it}y_{it}$  for each dynasty  $i$  in each period  $t$  satisfies  $\tau_{it}y_{it} = y_{it}$ , implying that no redistribution occurs. The next proposition shows that the expected income and intensity-of-effort levels of each dynasty  $i$ , in log form, converge over time to stationary limiting values under a libertarian tax policy.

**PROPOSITION V.3:** *Suppose that government in period 1 implements a permanent libertarian tax policy  $\tau = 0$ , known to all agents in all dynasties. Then for each dynasty*

$i$ ,

$$\lim_{t \rightarrow \infty} E[x_{it} | s_1] = [I - M']^{-1} d_i, \quad (33)$$

where  $M'$  denotes the transpose of the matrix  $M$  appearing in relation (25), and the  $2 \times 1$  vector  $d_i$  has the form

$$d_i = (\log(C) + mE[\log(\alpha_i)], -\log(D))' \quad (34)$$

for certain constant terms  $C$  and  $D$ . If the stationary probability distributions  $f_i$  and  $f_j$  that govern ability levels for dynasties  $i$  and  $j$  in each period  $t$  are identical, then  $d_i = d_j$ .

In the case of an egalitarian tax policy  $\tau = 1/N$ , it follows from (19) that the after-tax income for each dynasty  $i$  in each period  $t$  satisfies

$$\tau_{it} y_{it} = \sum_{j=1}^N y_{jt} / N \equiv \bar{y}_t, \quad (35)$$

implying that each dynasty receives an equal share of total social income. The next proposition provides a partial characterization of long-run outcomes for this case. More precisely, given the assumption that the expected values for both social income and individual dynasty incomes in log form converge over time to finite stationary values, analytical expressions are derived for long-run expected dynasty incomes and intensity-of-effort levels in log form.

**PROPOSITION V.4:** *Suppose that government in period 1 implements a permanent egalitarian tax policy  $\tau = 1/N$ , known to all agents in all dynasties. Suppose, also, that for each dynasty  $i$ ,*

$$\lim_{t \rightarrow \infty} E[\log(\bar{y}_t) | s_1] = K_i + \lim_{t \rightarrow \infty} E[\log(y_{it}) | s_1] \quad (36)$$

for some constant  $K_i$ , where the limits in (36) are finite valued. Then for each dynasty  $i$ ,

$$\lim_{t \rightarrow \infty} E[x_{it} | s_1] = [I - M']^{-1} h_i, \quad (37)$$

where  $M'$  denotes the transpose of the matrix  $M$  appearing in relation (25), and the components of the  $1 \times 2$  vector  $h'_i = (h_{1i}, h_{2i})'$  take the form

$$h_{1i} = \log(C) + mE[\log(\alpha_i)] - bv \log(N) + [u + av]K_i ; \quad (38)$$

$$h_{2i} = -\log(D) + b \log(N) - aK_i . \quad (39)$$

The constant terms  $C$  and  $D$  in (38) and (39) are the same as in (34).

It then follows as a corollary to Proposition V.4 that, under an egalitarian tax policy, the expected long-run marginal return to schooling decreases for each dynasty child as the total number of dynasties increases.

**COROLLARY V.2:** *Suppose the hypotheses of Proposition V.4 hold. Then the expected long-run marginal return to schooling effort in log form for each dynasty  $i$  is a strictly decreasing function of the total number  $N$  of dynasties, of the form*

$$\lim_{t \rightarrow \infty} E[\log(r_{it}^e) | s_1] = L_i - R \log(N) , \quad (40)$$

where the constants  $L_i$  and  $R > 0$  are independent of  $N$ .

Given (36), the following proposition establishes that each dynasty in the long run can expect to be worse off under pure egalitarianism than under pure libertarianism. Moreover, the greater the number of dynasties, the greater is the extent to which the dynasties are worse off under egalitarianism.

**Proposition V.5:** *Suppose condition (36) holds. Then for all sufficiently large  $N$  the expected income and intensity-of-effort levels in log form for each of the  $N$  dynasties are strictly lower in the long run under the egalitarian tax policy  $\tau = 1/N$  than under the libertarian tax policy  $\tau = 0$ ; and this discrepancy increases with increases in  $N$ .*

As will be seen in section VI, our simulation results suggest that the regularity condition (36) is indeed satisfied for the illustrative Basic Economy. Averaged across runs in log form,

social income levels, dynasty income levels, and dynasty intensity-of-effort levels all appear to converge to stationary limiting values under the egalitarian tax policy. Moreover, the expected income, effort, and social welfare levels achieved under the egalitarian tax policy fall well short of the levels achieved under the libertarian tax policy, even for  $N = 2$ .

The final proposition of this section provides another interesting check on the validity of the simulation results reported below in section VI. If average dynasty income constitutes a sufficiently good approximation for the true expected dynasty income for any dynasty  $i$  in any period  $t$ , then the true expected tax/subsidy level  $\tau_{it}$  for dynasty  $i$  in period  $t$  is a strictly increasing function of the tax policy  $\tau$ . That is, the more egalitarian the government's tax policy, the higher the subsidy level (or the lower the tax level) that dynasty  $i$  can expect to receive (or to pay).

**PROPOSITION V.6:** *Suppose that average dynasty income in some period  $t$  closely approximates the true expected dynasty income for dynasty  $i$  in period  $t$ , conditional on the period 1 state vector  $s_1$ . That is, suppose*

$$\sum_{j=1}^N y_{jt}/N \approx E[y_{it} | s_1] \quad (41)$$

for some  $i \in \{1, \dots, N\}$ . Then

$$\frac{\partial E[\tau_{it} | s_1]}{\partial \tau} > 0. \quad (42)$$

## VI. Simulation Results

The illustrative Basic Economy outlined in section IV is too complex to allow a detailed analytical characterization of the economy's responses to changes in the tax policy  $\tau$ . Simulation experiments were therefore conducted to determine how different tax policies influence economic growth and human capital investment in this economy.

To operationalize the model, specifications are needed for the number of dynasties, the distribution of abilities, and the parameters characterizing the utility and income functions. The number of dynasties  $N$  was first set at 2. Ability  $\alpha$  was assumed to be distributed uniformly over the range  $(0, 1)$ , implying a mean ability of .5. The value of  $\gamma$ , the exponent on current consumption in the utility function (16), was set to .5. The parameters of the income function (14) were set at  $\lambda = .5$ ,  $m = .5$ ,  $u = .5$ , and  $v = .5$ . Since  $\lambda$  serves as a scaling variable, we can generate proportionally larger or smaller measures of income without altering the relative levels of the variables by rescaling the value of  $\lambda$ .

Various parameterizations were tried to determine the sensitivity of the qualitative results to changes in the model parameters. The qualitative results proved to be quite robust to modifications in the parameters.

Each simulation was replicated 40 times over a 40 generation horizon. In each run, each dynasty received 40 independent draws from the uniform ability distribution, one draw per generation. For all runs, the starting values for parental income and schooling effort were initialized to be one. First, the purely libertarian tax policy ( $\tau = 0$ ) was run. For each dynasty in each year, the simulation generated values for income, consumption, schooling effort, and human capital investment. Then, holding fixed the sequence of ability draws, the simulation was rerun with progressively more egalitarian tax policies, ending with the purely egalitarian tax policy ( $\tau = N/2 = .5$ ). For each tax policy  $\tau$ , we also measured after-tax income and the tax/subsidy position of each dynasty. Finally, gross national products were computed by summing across dynasties at a point in time. Statistical summaries of the simulations are reported in Tables 1-4.

Table 1 contains the results from our base run. For each variable, we report the mean and standard deviation taken over 40 replications of 39 generations of generated data. The initial generation was deleted since all runs had the same initial levels of income and effort. For the two-dynasty model, this implies 3120 observations ( $39 \times 40 \times 2$ ). The simple correlation



between current and once-lagged variables are also reported to determine the extent to which parent and child outcomes are correlated.

—Table 1 About Here—

Immediately apparent from Table 1 is that tax policies can increase human capital production, as in the model by Loury (1981). With two dynasties, the tax policy  $\tau$  can vary from 0 to .5. A relatively small tax policy ( $\tau = .04$ ) increases income and parental investment in children. It also reduces the variance in those parental investments across households. By transferring income toward poorer households, poorer parents are able to invest in the education of their high ability children. This reduces the inefficiency in human capital investment caused by the relative overinvestment in the children of high income parents and the relative underinvestment in children of low income parents. As a result, the economy as a whole benefits. Moreover, introduction of the tax policy begins to reduce the variance in after-tax income. The reduction in uncertainty about future income and consumption is a second benefit from the tax policy.

The result that tax policies can increase expected GNP is in marked contrast to the implications of the model of King and Rebelo (1990), who found that increasing taxes uniformly reduced GNP growth. Nevertheless, Table 1 also indicates that a moderate tax policy dominates relatively more egalitarian tax policies. Although transfers allow poorer parents to invest more in their children, the income taxes used to finance the transfers also lower the expected marginal returns to these investments for all children. To see this, note that an increase in  $\tau$  decreases the child's expected marginal return to schooling effort in equation (21). As children see the expected marginal return from their human capital investments declining, they apply less effort to their schooling. This reduction in effort begins at relatively modest tax policy levels, and effort continues to decline as the tax policy becomes progressively more egalitarian. In the limit, schooling effort under the perfectly egalitarian

tax policy  $\tau = .5$  is only 55 percent of the level under the libertarian tax policy  $\tau = 0$ . As a result, the GNP attained under a policy that guarantees income equality averages just over one-half the level of GNP attained under a policy that involves no income redistribution.<sup>6</sup> These findings are consistent with the predictions of Proposition V.5.

Table 1 also tells how a parent's income is related to his child's income. As tax policies become more egalitarian, the correlation between the before-tax income of a parent and child falls monotonically. The same is not true of after-tax income. The reduction in after-tax income correlation across generations is more moderate. As  $\tau$  rises from 0 to .15, the correlation coefficient declines by .45 for before-tax income, but by only .19 for after-tax income. Thereafter, while intergenerational income correlations continue to fall for before-tax income, they begin to rise for after-tax income. In the limit, the intergenerational after-tax income correlation under the perfectly egalitarian tax policy  $\tau = .5$  is larger than under the libertarian tax policy  $\tau = 0$ . The depressing effect of the perfectly egalitarian tax policy on human capital investment reduces the rate of growth and the variance in GNP to such an extent that it increases the intergenerational correlation in incomes. Since GNP is shared equally by all households, the intergenerational correlation in dynasty after-tax income is exactly the same as the intergenerational correlation in GNP.

By lowering incentives to invest in human capital, the tax policy  $\tau$  also increases the relative importance of the ability draw in determining relative pre-tax income across dynasties. As a result, the intergenerational correlation in dependency (as measured by the autocorrelation of  $\tau_{it}$ ) declines as the tax policy becomes more egalitarian. Since  $\tau_{it}$  is above 1 for subsidized households and below 1 for taxed households, the decline in the intergenerational correlation implies that children from relatively poorer households have a higher probability of receiving above average pre-tax earnings under more egalitarian tax policies. Of course,

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<sup>6</sup>Recall from section V that, even under a perfectly egalitarian tax policy, each parent will still desire to invest a positive amount of resources in his child's education as long as the parent's after-tax income is positive.

dependency is measured relative to average earnings in the economy, and not in any absolute sense. As such, the low-income households in the relatively less-egalitarian economies will still earn more on average than the higher-income households under the perfectly egalitarian regime.

Proposition V.6 predicts that dynasties will expect to receive a higher subsidy (or lower tax) from the government as the tax policy becomes more egalitarian. This is clearly borne out in the simulations. The expected value of  $\tau_{it}$  rises from 1.0 to 1.15 as  $\tau$  increases from 0 to .5.

One last outcome from the simulations in Table 1 is that households allocate about 39 percent of their after-tax income to their children and the remainder to consumption. Olson (1983, p. 40) estimated for the United States that two-parent families with two children born in 1980 would allocate about 37 percent of their income to raising their children to age 22. Thus, the parameterizations in Table 1 yield reasonable estimates of parental resource allocations.

It is important to determine the sensitivity of the conclusions derived thus far to changes in the parameters. Simulations were run assuming different values of the parameters of the utility function, the income function, and the number of dynasties. In Table 2, we lower the utility function weight  $\gamma$  on current consumption to .25, thereby raising the utility function weight  $[1 - \gamma]$  on future consumption to .75. As a result, parents increase their investment in their children and the economy grows to almost five times the levels in Table 1. However, none of the qualitative implications change relative to those derived from Table 1. Small tax policies raise expected income and lower the variance in income, but progressively more egalitarian tax policies ultimately lower income, GNP, schooling effort, and human capital investment. Initially, taxes lower the intergenerational correlation in after-tax income, but ultimately the highest intergenerational correlations are found under the perfectly egalitarian tax policy. The share of income devoted to children rises to 59 percent, well above the actual

national average in the United States.

—Table 2 About Here—

In Table 3, the utility parameters are the same as in Table 1, but we increase the income elasticity (i.e., the elasticity of a child's income) with respect to parental human capital investment from  $u = .5$  to  $u = .75$  and lower the income elasticity with respect to schooling effort from  $v = .5$  to  $v = .25$ . This change in parameterization leaves the income elasticity with respect to the scale factor  $\lambda$  unchanged at 1; see equation (14). Comparing Table 3 with Table 1, tax policies involving increased income transfers now initially have a relatively larger positive effect on dynasty incomes and on GNP, and the adverse effects of income transfers on these variables do not outweigh the positive effects until later in the progression toward more egalitarian tax policies. Nonetheless, the general Table 1 pattern of results still holds. Perfectly egalitarian tax policies are still dominated by purely libertarian tax policies, and the intergenerational correlation in after-tax income is still highest under the egalitarian tax policy. The share of household income devoted to children is 45 percent, a bit high relative to the 37 percent reported by Olson (1983).

—Table 3 About Here—

As the number of dynasties increases, GNP growth should be smoothed since extreme individual ability draws will have a smaller effect on national income. This is shown clearly in Table 4. The coefficient of variation in GNP is about two-thirds the level in Table 1. However, the increase in the number of dynasties also reduces expected marginal returns to schooling effort, consistent with Corollary V.2. Individuals have a greater incentive to free ride on the economy as tax policies become more egalitarian. In the perfectly egalitarian regime, schooling effort falls to .64, only 43 percent of the schooling effort in the perfectly egalitarian two-dynasty economy in Table 1. The free-rider problem is so great under the perfectly egalitarian tax policy that GNP with five dynasties is not much larger than with two

dynasties, and dynasty average income is less than half that in the two-dynasty economy. Increasing the number of dynasties exacerbates the disincentive effects of egalitarian tax policies.

—Table 4 About Here—

All of the statistics reported thus far have looked at the generated data over the entire trajectory of 40 generations. The analytical results reported in section V predict that these data values should converge in expectation to steady state levels under each of the two tax policy extremes, libertarianism and egalitarianism. In Figure 2, the time path of average GNP is graphed for six tax policies spanning these two extremes. In each case the time paths appear to level off after fifteen to twenty generations. The time paths show very similar GNP levels for the three most libertarian tax policies, and then show progressively lower GNP levels for the remaining three most egalitarian tax policies.

—Figure 2 About Here—

Another method for assessing the various tax policies is to measure the expected lifetime utility levels associated with these policies. By relation (17), the log of the true lifetime utility  $U_{i1}$  achieved by a dynasty  $i$  parent in period 1 can be expressed as an infinite sum in log form of all of the instantaneous utilities achieved by dynasty  $i$  parents in periods  $t \geq 1$ . We constructed a truncated approximation for this infinite sum by summing over the instantaneous utilities achieved by dynasty  $i$  parents in generations fifteen through thirty-five only. This approximation was exponentiated to reclaim an estimate  $\hat{U}_{i1}$  for  $U_{i1}$ . The expected lifetime utility of a period 1 parent under a given tax policy  $\tau$  was then estimated by first averaging the estimates  $\hat{U}_{i1}$  across dynasties  $i = 1, \dots, N$  for each of forty replications of the economy under the tax policy  $\tau$ , and then further averaging these averages over the forty runs. To check the robustness of our expected lifetime utility estimate, we also calculated a second estimate by repeating the same procedure using a truncated approximation which

included the instantaneous utilities achieved by dynasty  $i$  parents in generations twenty through forty only.

Table 5 reports these estimated expected lifetime utilities for a number of different income, effort, and utility parameter specifications. These results generally mimic the results reported above for the simple GNP measure of welfare. For each reported parameter specification, the perfectly libertarian tax policy ( $\tau = 0$ ) yields higher welfare than the perfectly egalitarian tax policy ( $\tau = .5$ ), but small positive tax policies dominate both of these extreme tax policies. The utility welfare measure does rank modestly egalitarian tax policies (e.g.,  $\tau = .08$ ) more highly than does a simple comparison of GNP. The reason is that positive tax policies lower uncertainty about income, even if they may slightly lower expected income. Small tax policies have bigger welfare-improving effects as the utility weight  $\gamma$  on current consumption decreases, and also as the income function parameter  $u$  increases, i.e., as parental human capital investments become more productive. Small tax policies become less welfare-enhancing as the number of dynasties increases.

## VII. Conclusion

This paper has examined the impact of redistributive income tax policies on human capital investment, per capita GNP, and social welfare in the context of a multi-dynasty overlapping generations economy. We find that modest redistributive tax policies can raise human capital investment, per capita GNP, and social welfare by increasing the human capital investments of relatively poor families in their children. Further welfare gains are obtained from the reduction in uncertainty regarding after-tax incomes that results from the implementation of the transfer program.

Nevertheless, the taxes which funds these transfers also lower children's perceptions of their marginal returns to schooling. The resulting adverse effects on schooling effort quickly overtake the beneficial effects of the transfers as the tax policy becomes progressively more

egalitarian. In the limit, a purely egalitarian tax policy lowers schooling effort to such a great extent that per capita GNP and social welfare fall well short of the levels achieved under a purely libertarian tax policy. These adverse effects on schooling effort worsen as the number of dynasties increases. Also, human capital investment and per capita GNP growth are more adversely affected in economies in which parents place a relatively large weight on current consumption, and in economies in which parental investments in their children's human capital are relatively less productive.

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